Amplitude Modulation

Manasvi Grover
Electronics and Communications Department
Northern India Engineering College, New Delhi, India

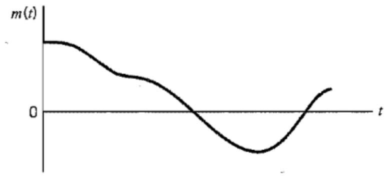
Maninder Bir Singh Gulshan
Electronics and Communications Department
Northern India Engineering College, New Delhi, India

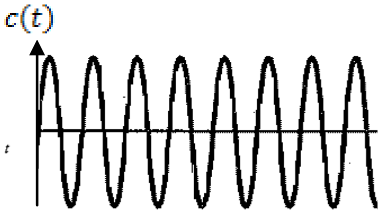
Amrita Kaul
Electronics and Communications Department
Northern India Engineering College, New Delhi, India

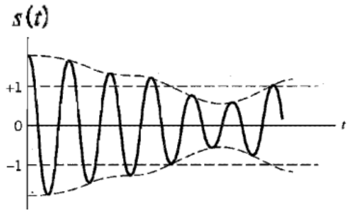
***Abstract*– Amplitude modulation is process of superimposing of message or baseband signal over carrier signal, so that signal will travel effectively fast and secure through long distance communication. In this paper, we will study about various methods of AM waves and their generations techniques. The main idea behind this paper is to compare power, spectrum, efficiency and modulation techniques of AM.**

I.INTRODUTION

Amplitude Modulation:
The maximum amplitude of carrier signal is made proportional to the instantaneous value of amplitude of modulating or baseband signal in system.
According to the definition,







There are three types of AM :

1. DSB-FC or AM(Double Sideband –Full Carrier)
2. DSB-SC(Single Sideband –Suppressed Carrier)
3. SSB-SC(Single Sideband –Suppressed Carrier)

II.MATHEMATICAL EXPRESSIONS

DSB-FC : Double Sideband – Full Carrier

Also know to be Amplitude Modulation.
Let a message signal or baseband signal be m(t) for single tone,
$\mathrm{m(t)} = \mathrm{A_m} \cos \omega_m t$ or $\mathrm{m(t)} = \mathrm{A_m} \sin \omega_m t$ ……(1)
And the carrier signal be c(t),
$c(\mathrm{t}) = \mathrm{A_c} \cos \omega_c t$ or $\mathrm{c(t)} = \mathrm{A_c} \sin \omega_c t$ ……..…(2)
The modulated signal be s(t),
$s(t) = A_c[1 + k_a m(t)] \cos \omega_c t$ ……………….(3)

Modulated signal s(t) is (1+$k_a m(t)$) times c(t).

where,
A_m= Amplitude of baseband signal (volts / V)
A_c= Amplitude of carrier signal (volts / V)
ω_m= Angular frequency of message signal (rad / s)
ω_c= Angular frequency of carrier signal (rad / s)
k_a= Amplitude sensitivity factor

Substituting eqn. (1) in eqn.(3),

$$s(t) = A_c[1 + k_a\ A_m\cos\omega_m t]\cos\omega_c t\ ,$$
$$= A_c\cos\omega_c t + k_a\ A_m A_c\cos\omega_m t\cos\omega_c t\ ,$$

$$= A_c\cos\omega_c t + \frac{k_a\ A_m A_c\cos(\omega_m+\omega_c)t}{2} + \frac{k_a\ A_m A_c\cos(\omega_m-\omega_c)t}{2} \qquad \because 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$= A_c\cos\omega_c t + \frac{\mu A_c}{2}\cos(\omega_m+\omega_c)t + \frac{\mu A_c}{2}\cos(\omega_m-\omega_c)t \qquad \because\ \mu = k_a A_m,$$ whereμ is Modulation Index.

$$= A_c\cos\omega_c t + \frac{\mu A_c}{2}\cos(\omega_m+\omega_c)t + \frac{\mu A_c}{2}\cos(\omega_m-\omega_c)t$$

$$as, \cos\omega_c t = \frac{e^{j\omega_c t}+e^{-j\omega_c t}}{2}\ ,$$

Power of DSB-FC or AM :

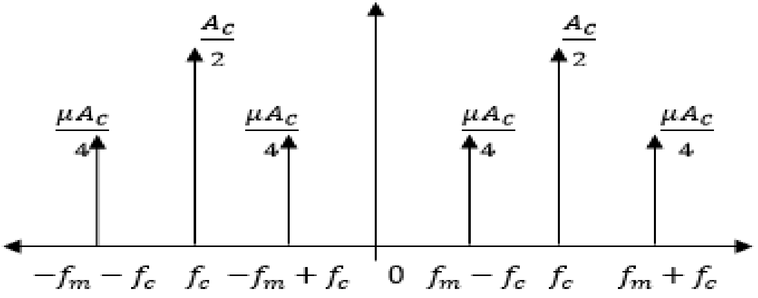
$P_{ac} = \frac{A^2}{2R}$, from the above eqn.

$$\boldsymbol{P_t = \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right]}.$$

Therefore ,

$$\boldsymbol{s(t) = \frac{A_c}{2}e^{j\omega_c t} + \frac{A_c}{2}e^{-j\omega_c t} + \frac{\mu A_c}{4}e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4}e^{-j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4}e^{j(\omega_c-\omega_m)t} + \frac{\mu A_c}{4}e^{-j(\omega_c-\omega_m)t}}$$

Frequency Spectrum (for single tone),



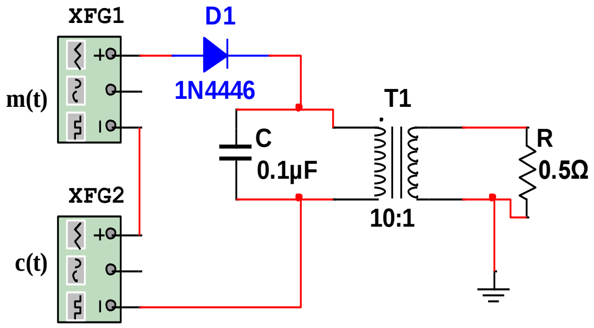
DSB-FCcontains (Single tone):

1. Carrier Signal c(t) at f_c.
2. One Upper Sideband at $f_c + f_m$.
3. One Lower Sideband at $f_c - f_m$.

Techniques for the Generation of DSB-FC are:

1. Square Law Modulator

The Square law modulator circuit contains:
(a). A non-linear device (SLD) – Diode
(b). A band pass filter (BPF)
(c). A carrier source and modulating signal
V_1 is voltage before D1 and V_2 is voltage after D1.
V_0 is output voltage across R.



$$V_0 = aV_1 + bV_1^2 + \cdots$$
$$V_1 = m(t) + c(t)\ ;\ V_1 = m(t) + A_c\cos\omega_c t$$
$$V_2 = aV_1 + bV_1^2\ ;$$
$$V_2 = am(t) + aA_c\cos\omega_c t + b[m(t) + A_c\cos\omega_c t]^2$$

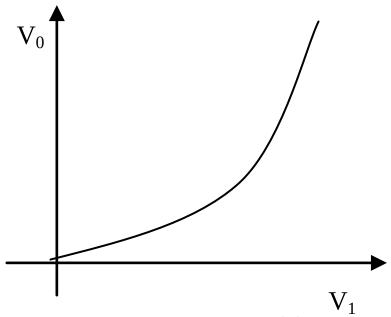
After passing through BPF,

$$V_2 = aA_c\cos\omega_c t + 2bm(t)\ A_c\cos\omega_c t$$

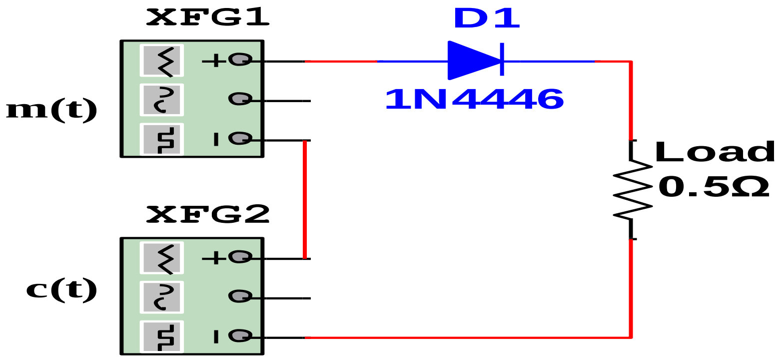
Comparing above eqn. with s(t),

$$\boldsymbol{\therefore V_0 = s(t) = aA_c[1 + \frac{2b}{a}m(t)]\cos\omega_c t}$$

Idealized input-output relation



2. Switching Modulator



Idealized input-output relation

$V_1 = m(t) + A_c \cos \omega_c t\,;\ V_2 = V_1(t).\, g_p(t)$

$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c t(2n-1)]$

$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t + odd\ harmonics$

$V_2 = V_1(t).\, g_p(t)$

$V_2 = (m(t) + A_c \cos \omega_c t).\, (\frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t)$

$V_2 = \frac{m(t)}{2} + \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos \omega_c t + \frac{2}{\pi} A_c\, (\cos\omega_c t)^2$

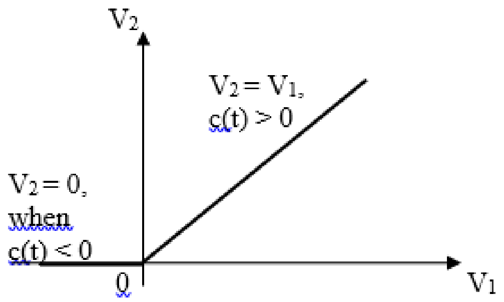
After passing through BPF,

$V_2 = \frac{A_c}{2} \cos \omega_c t + \frac{2}{\pi} m(t) \cos 2\pi f_c t$

Comparing above eqn. with s(t),

$$\therefore \boldsymbol{V_2 = s(t) = \frac{A_c}{2}[1 + \frac{4}{\pi A_c} m(t)] \cos \omega_c t}$$

where, $k_a = \frac{4}{\pi A_c}$.



Idealized input-output relation

DSB-SC : Double Sideband – Suppressed Carrier

Let a message signal or baseband signal be m(t) for single tone,
$m(t) = A_m \cos \omega_m t$ or $m(t) = A_m \sin \omega_m t$ ……………………………………………………………(1)
No carrier signal as it is suppressed.
The modulated signal be s(t),
$s(t) = A_c[k_a m(t)] \cos \omega_c t$ …………………………………………………….……(2)

Modulated signal s(t) is $k_a m(t)$ times c(t).

where,
A_m = Amplitude of baseband signal (volts / V)
A_c = Amplitude of carrier signal (volts / V)
ω_m = Angular frequency of message signal (rad / s)
ω_c = Angular frequency of carrier signal (rad / s)
k_a = Amplitude sensitivity factor
Substituting eqn. (1) in eqn.(2),
$s(t) = A_c[k_a A_m \cos \omega_m t] \cos \omega_c t = k_a A_m A_c \cos \omega_m t \cos \omega_c t$

$= \frac{k_a A_m A_c \cos(\omega_m + \omega_c)t}{2} + \frac{k_a A_m A_c \cos(\omega_m - \omega_c)t}{2}$, $\qquad \because 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$= \frac{\mu A_c}{2} \cos(\omega_m + \omega_c)t + \frac{\mu A_c}{2} \cos(\omega_m - \omega_c)t$ $\qquad \because \mu = k_a A_m$, where μ is Modulation Index

as, $\cos \omega_c t = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$,

Power of DSB-SC:

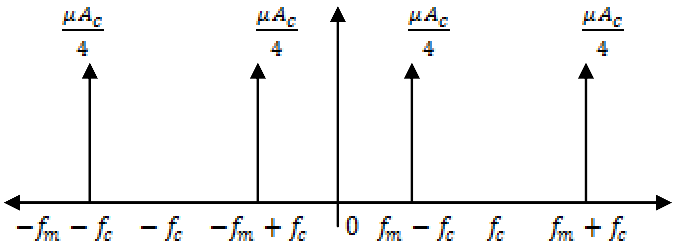
$P_{ac} = \frac{A^2}{2R}$, from the above eqn.

$P_t = \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = \frac{\mu^2 A_c^2}{4R}$.

Therefore ,

$$s(t) = \frac{\mu A_c}{4} e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{j(\omega_c-\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c-\omega_m)t}$$

Frequency Spectrum (for single tone),



DSB-SC contains (Single tone):

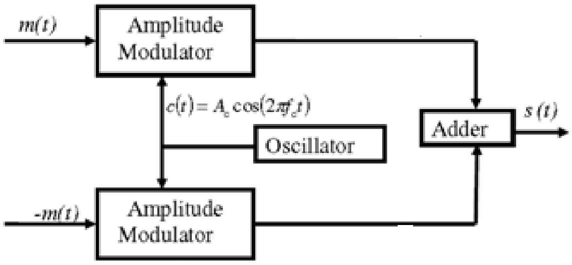
1. One Upper Sideband at $f_c + f_m$.
2. One Lower Sideband at $f_c - f_m$.

Techniques for the Generation of DSB-SC are:

1. Balanced Modulator

The balanced modulator is used to suppress the unwanted carrier in an AM wave. The carrier and modulating signals are applied to the inputs of the balanced modulator and we get the DSB signal with suppressed carrier at the output of the balanced modulator.

Block diagram of Balanced Modulator



$$s'(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

$$s''(t) = A_c[1 - k_a m(t)] \cos \omega_c t$$

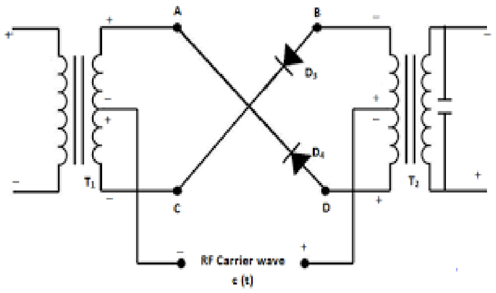
$$s(t) = s'(t) - s''(t) ;$$

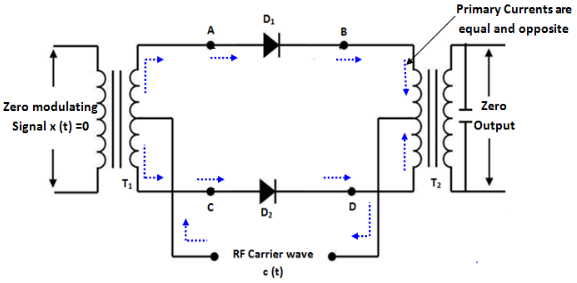
$$\boldsymbol{s(t) = 2k_a A_c \cos \omega_c t}$$

where, $s'(t)$ and $s''(t)$ are the output signals of respective AM's and $s(t)$is output for DSB-SC.

2. Ring Modulator or Chopper Modulator

Ring modulation is a signal-processing function in electronics, an implementation of amplitude modulation or frequency mixing, performed by multiplying two signals, where one is typically a sine wave or another simple waveform.





SSB-SC : Single Sideband – Suppressed Carrier

An AM signal with no carrier and one sideband is called a single sideband. The upper and lower sidebands contain the same information, and is not preferred over the other.
Let a message signal or baseband signal be m(t) for single tone,
$m(t) = A_m \cos \omega_m t \text{ or } m(t) = A_m \sin \omega_m t$ ……………………………………………..……(1)
No carrier signal as it is suppressed.
The modulated signal be s(t),
$s(t) = A_c[k_a m(t)] \cos \omega_c t$ ……………………………………………(2)

Modulated signal s(t) is $k_a m(t)$ times c(t).
where,
A_m= Amplitude of baseband signal (volts / V)
A_c= Amplitude of carrier signal (volts / V)
ω_m= Angular frequency of message signal (rad / s)
ω_c= Angular frequency of carrier signal (rad / s)
k_a= Amplitude sensitivity factor
Substituting eqn. (1) in eqn.(2),
$s(t) = A_c[k_a A_m \cos \omega_m t] \cos \omega_c t = k_a A_m A_c \cos \omega_m t \cos \omega_c t$

$= \frac{k_a A_m A_c \cos(\omega_m+\omega_c)t}{2} + \frac{k_a A_m A_c \cos(\omega_m-\omega_c)t}{2}$, $\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)$
$= \frac{\mu A_c}{2}\cos(\omega_m + \omega_c)t + \frac{\mu A_c}{2}\cos(\omega_m - \omega_c)t$ $\because \mu = k_a A_m$, where μ is Modulation Index
as, $\cos \omega_c t = \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}$,

After passing through Band Pass Filter either $f_m + f_c$ will pass or $f_m - f_c$ will pass ,

$s(t) = \frac{\mu A_c}{2}\cos(\omega_m + \omega_c)t$ OR $s(t) = \frac{\mu A_c}{2}\cos(\omega_m - \omega_c)t$

Power of SSB-SC:

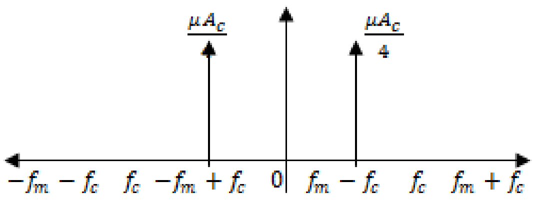
$P_{ac} = \frac{A^2}{2R}$, from the above eqns.

$P_t = \frac{\mu^2 A_c^2}{8R}$.

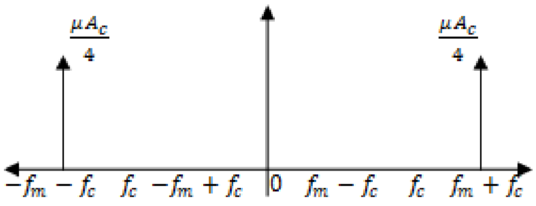
Therefore ,

$s(t) = \frac{\mu A_c}{4} e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c+\omega_m)t}$ OR $s(t) = \frac{\mu A_c}{4} e^{j(\omega_c-\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c-\omega_m)t}$

Frequency Spectrum (for single tone).



For Lower Sideband.



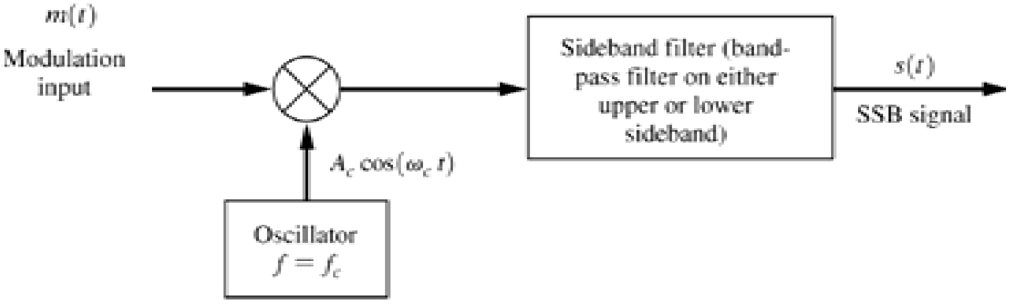
For Upper Sideband.

SSB-SC contains (Single tone):
One Upper Sideband at $(f_c + f_m)$ OR One Lower Sideband at $(f_c - f_m)$

Techniques for the Generation of SSB-SC are:

1. Frequency Discrimination Method

One way to generate SSB is to first generate DSB-SC and the suppress the side bands by filtering.



(Derived from DSB-SC)
After multiplying $m(t)$and $c(t)$,

$$s(t) = \frac{\mu A_c}{4} e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{j(\omega_c-\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c-\omega_m)t}$$

After passing through the Band Pass Filter, we get,
For Upper Sideband,

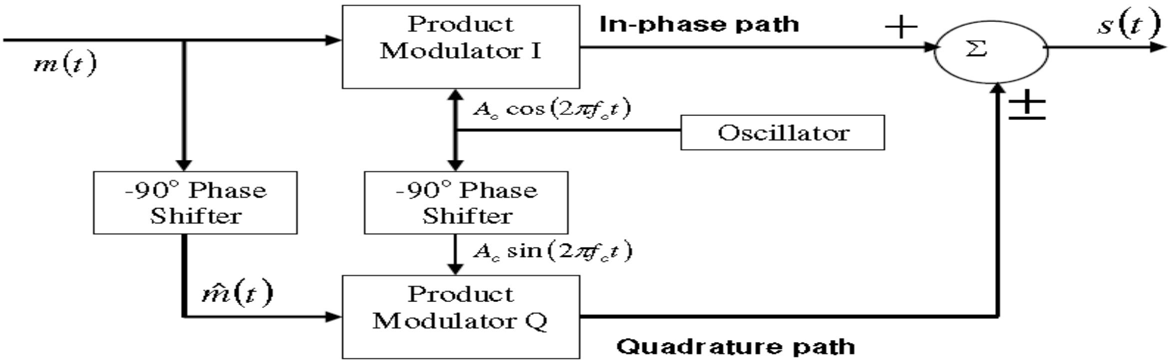
$$s(t) = \frac{\mu A_c}{4} e^{j(\omega_c+\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c+\omega_m)t}$$

For Lower Sideband,

$$s(t) = \frac{\mu A_c}{4} e^{j(\omega_c-\omega_m)t} + \frac{\mu A_c}{4} e^{-j(\omega_c-\omega_m)t}$$

2. Phase Discrimination Method

It is possible to generate SSB by proper phasing of signals. The method essentially uses two product modulators to create two DSB-SC signals that are phase shifted by 90°. The two DSB signals are then combined to cancel one of the side bands.



REFERENCES
[1] Dr. Sanjay Sharma, *Communication Systems*, 6th Edition: 2016, pp. 197
[2] Simon Haykin, *Communication Systems*,4th Edition: United States of America, pp. 90, 91